Estimating Absorption Coefficient from a Single Image via Entropy Minimization

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Abstract

When light passes through a liquid, its energy is attenuated due to absorption. The attenuation depends on both the spectral absorption coefficient of a liquid and the optical path length of light, and is described by the Lambert-Beer law. The spectral absorption coefficients of liquids are often unknown in real-world applications and to be measured/estimated in advance, because they depend not only on liquid media themselves but also on dissolved materials. In this paper, we propose a method for estimating the three-band (RGB) absorption coefficient of a liquid only from a single color image of an under-liquid scene taken from the outside of the liquid in a passive and non-contact manner. Specifically, our proposed method investigates the observed colors in the log of chromaticity band-ratio space, and estimates the absorption coefficient up to a certain ambiguity via entropy minimization. Moreover, we reveal the effects of the ambiguity on the applications to under-liquid image/scene analysis. We conducted a number of experiments using real images, and confirmed that our method works well and is useful for under-liquid shape recovery, absorption removal, and reflectance recovery.

1 Introduction

When light passes through a liquid, a part of the light is often absorbed and scattered by the liquid, and then its energy is attenuated in general. For transparent liquids with negligible scattering such as colored water, it is known that the attenuation of light energy due to absorption is described by the Lambert-Beer law [15]; it depends on both the *spectral absorption coefficient* of a liquid and the optical path length of light in the liquid.

The absorption due to liquid is an important clue to solving computer vision problems; shape recovery of underwater objects [3, 12, 13, 18] and liquid detection on unknown surfaces [20, 21] are achieved under the assumption that the spectral absorption coefficients of liquids are known. Unfortunately, however, the absorption coefficients of liquids are often unknown in real-world applications because they depend not only on liquid media themselves but also on dissolved materials. Therefore, the absorption coefficients of liquids are to be measured/estimated in advance.

Conventionally, the spectral absorption coefficients of liquids are measured via absorption spectroscopy [7, 11] in an *active and contact* manner. Specifically, when the Spectral Power Distributions (SPDs) of the light both before and after transmitting a liquid of interest are known, its spectral absorption coefficient is derived from the logarithm of the ratio of those SPDs on the basis of the Lambert-Beer law. In the community of computer vision, the three-band (RGB) absorption coefficient or attenuation coefficient, *i.e.* the summation of the absorption and scattering coefficients, of a liquid is often measured or estimated. It is known that the absorption/attenuation coefficient can be estimated from an image of a known calibration target at known distances [2, 14, 19] and multiple images of the same object located at different distances [4, 8, 9, 17, 22].

In contrast to those existing methods, we propose a method for estimating the three-band absorption coefficients of liquids in a *passive and non-contact* manner. Our proposed method estimates the absorption coefficient of a liquid only from a single color image of an under-liquid scene taken from the outside of the liquid. Therefore, our method does not require the SPD of the light before transmitting the liquid [7, 11], calibration targets [2, 14, 19], or distance measurement/estimation [4, 8, 9, 17, 22].

The difficulties of estimating the absorption coefficient of a liquid from a single color image are in the entanglement that the RGB value observed at a point on an object surface depends not only on the absorption coefficient of the liquid but also on the geometry (surface normal and depth) and reflectance of the surface point and the color of a light source. In other words, it is difficult to estimate how much the observed color is caused by the absorption and the object surface. The key idea of our proposed method is to disentangle those properties by investigating the observed colors in the log of chromaticity band-ratio space [6]. Specifically, we theoretically show that the pixel values at surface points with the same reflectance distribute along a straight line in the space, and the slope of the line is described by the three-band absorption coefficient. Then, our method computes the slope of the line via entropy minimization [6] and estimates the three-band absorption coefficient up to a certain ambiguity.

Moreover, we reveal the effects of the ambiguity on the applications to under-liquid shape recovery, absorption removal, and reflectance recovery, and show that the absorption coefficient is useful for those applications even though it has the ambiguity. We conducted a number of experiments using real images, and confirmed that the proposed method works well for real images and is useful for under-liquid image/scene analysis.

The main contributions of this paper are threefold. First, we propose a novel method for estimating the three-band absorption coefficient of a liquid in a passive and non-contact manner. The proposed method achieves the estimation of the absorption coefficient only from a single color image without requiring calibration targets and distance measurement/estimation. Second, we show the ambiguity in the three-band absorption coefficient estimated by using our method. In addition, we reveal the effects of the ambiguity on the applications to under-liquid image/scene analysis. Third, we conducted a number of experiments using real images, and confirmed that our method works well and is useful for under-liquid shape recovery, absorption removal, and reflectance recovery.

2 Related Work

2.1 Absorption Measurement

Absorption spectroscopy [7, 11] is a classical method for measuring the spectral absorption coefficient of a liquid of interest in an active and contact manner. When the SPDs of the light both before and after transmitting the liquid are known, its spectral absorption coefficient is derived from the logarithm of the ratio of those SPDs on the basis of the Lambert-Beer law [15]. In the computer vision problems such as shape recovery [3, 12, 13, 18] and liquid detection [20, 21], however, the SPD (or color) of the light before transmitting a liquid, *i.e.* the spectral radiance on an object surface under the liquid is unknown. Therefore, we achieve the estimation of the absorption coefficient of a liquid only from a single image without requiring the color of the object surface by investigating the observed colors in the log of chromaticity band-ratio space.

In the community of computer vision, the three-band (RGB) attenuation coefficient, *i.e.* the summation of the absorption and scattering coefficients of a liquid is often measured or estimated. It is known that the attenuation coefficient can be estimated from an image of a known calibration target at known distances [1, 14, 19], but such estimation requires external hardware and distance measurement. The attenuation/absorption coefficient can be estimated from multiple images of the same object located at different distances; the distances are assumed to be known [22], or measured by a sonar [9, 17], or recovered via structure from motion [2, 8]. In contrast, our proposed method estimates the three-band absorption coefficient only from a single color image with neither a known calibration target nor known distances nor geometric calibration. Recently, Kageyama *et al.* [10] propose a method for estimating the spectral absorption coefficient in a passive and non-contact manner, but it requires two hyperspectral images taken from different viewpoints.

2.2 Computer Vision Applications

The absorption due to liquid is an important clue to shape recovery. Asano *et al.* [3] make use of the fact that water absorbs Near InfraRed (NIR) light [5], and show that the shape (depth) of an under-water scene can be recovered from two single-view images at different wavelengths in NIR range. Takatani *et al.* [18] extends the above method by using an event-based camera with temporally modulated illumination, and then achieve robust shape reconstruction in water. Murai *et al.* [13] reconstruct both the surface normals and depth of a dynamic object in water by using multi-directional NIR lighting. Furthermore, Kuo *et al.* [12] achieve shape reconstruction of a dynamic and non-rigid object in water.

The absorption due to liquid is useful also for liquid detection. Wang *et al.* [21] make use of the fact that the absorption due to water decreases the apparent spectral reflectance on an object surface. They achieve per-pixel water detection on surfaces with unknown reflectance by using the low-dimensional linear model of spectral reflectance from visible to NIR wavelengths. Further, Wang and Okabe [20] extend their method to water and oil detection on unknown surfaces by simultaneously estimating the types of liquids and optical path lengths.

The above applications assume that the spectral absorption coefficients of liquids of interest (water and oils) are known. Unfortunately, however, the absorption coefficients of liquids are often unknown in real-world applications and are to be measured/estimated in advance. The three-band absorption coefficients estimated by using our proposed method in a passive and non-contact manner are effective for under-liquid image/scene analysis.

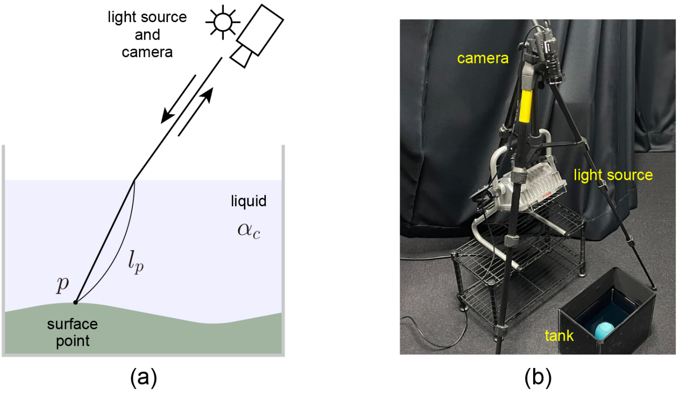

Figure 1: Our setup for absorption estimation: (a) a sketch and (b) a prototype.

3 Proposed Method

3.1 Log of Chromaticity Band-Ratio Space

We propose a method for estimating the three-band (RGB) absorption coefficient of a liquid of interest in a passive and non-contact manner. As shown in Figure 1(a), our proposed method uses a single color image of an under-liquid scene taken with a camera outside the liquid under a point light source located at the same position as the camera. We assume that the liquid is spatially uniform and transparent with negligible scattering [1] and that the attenuation of light energy due to absorption obeys the Lambert-Beer law [15].

When we observe an under-liquid scene from the outside of the liquid, the pixel value i_{pc} of the c-th color channel ($c = R, G, B$) at the p-th pixel ($p = 1, 2, 3, ..., P$) is given by

$$i_{pc} = g_p s_c r_{pc} e^{-2\alpha_c l_p} \tag{1}$$

according to the Lambert-Beer law. Here, g_p, s_c, r_{pc}, α_c, and l_p are the geometric term[2] depending on the surface normal and the light source direction, the intensity of the light source, the reflectance of the surface point, the absorption coefficient of the liquid, and the depth (optical path length) of the surface point respectively. Note that we consider round-trip depth $2l_p$ since the location of the light source is the same as that of the camera.

In order to cancel out the geometric terms, we take the ratio of the color channels i_{pG} and i_{pR} for example as

$$\frac{i_{pG}}{i_{pR}} = \frac{s_G r_{pG}}{s_R r_{pR}} e^{-2(\alpha_G - \alpha_R) l_p}. \tag{2}$$

Then, in order to extract the (difference of) the absorption coefficient, we take the logarithm of the color channel-ratio as

$$\ln \frac{i_{pG}}{i_{pR}} = \ln \frac{s_G r_{pG}}{s_R r_{pR}} - 2(\alpha_G - \alpha_R) l_p. \tag{3}$$

[1] Note that our target is not underwater images, where scattering cannot be ignored in real scenarios, but colored-transparent liquid images.

[2] The Fresnel term is absorbed into the geometric term.

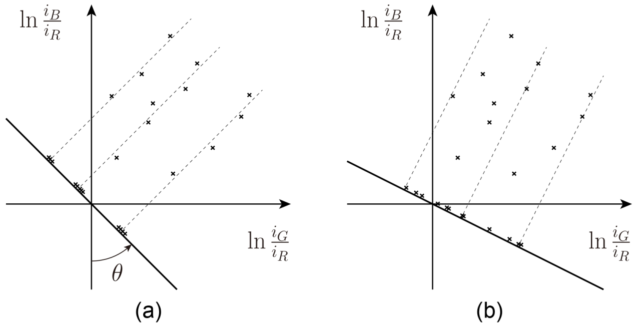

Figure 2: The distribution of the observed values in the log of chromaticity band-ratio space and their projection to the 1-D line: (a) the direction with minimal entropy and (b) the other direction with larger entropy.

Considering the ratio of the color channels i_{pB} and i_{pR} in the same manner, the relationship between the absorption coefficient and the other variables is given by

$$\begin{pmatrix} \ln \frac{i_{pG}}{i_{pR}} \\ \ln \frac{i_{pB}}{i_{pR}} \end{pmatrix} = \begin{pmatrix} \ln \frac{s_G r_{pG}}{s_R r_{pR}} \\ \ln \frac{s_B r_{pB}}{s_R r_{pR}} \end{pmatrix} - 2 l_p \begin{pmatrix} \alpha_G - \alpha_R \\ \alpha_B - \alpha_R \end{pmatrix}. \quad (4)$$

The 2-dimensional space of $\ln(i_G/i_R)$-$\ln(i_B/i_R)$ is called the *log of chromaticity band-ratio space* [6], and used for shadow removal under black body radiation.

As shown in Figure 2, eq.(4) tells that the observed values $(\ln(i_{pG}/i_{pR}), \ln(i_{pB}/i_{pR}))^\top$ of the surface points with the same reflectance r_{pc} but the different depths l_p distribute along a straight line in the log of chromaticity band-ratio space. The straight line passes through the point $(\ln(s_G r_{pG}/s_R r_{pR}), \ln(s_B r_{pB}/s_R r_{pR}))^\top$ and linearly extends to the direction $(\alpha_G - \alpha_R, \alpha_B - \alpha_R)^\top$ according to the depth l_p. Therefore, if the surface points of an under-liquid scene have multiple reflectances (colors), the distribution of the observed values in the log of chromaticity band-ratio space consists of parallel lines. This is because those lines share the same direction, *i.e.* the second term of the right-hand side of eq.(4).

3.2 Entropy Minimization

As discussed in Section 3.1, the estimation of (the difference of) the three-band absorption coefficient results in estimating the slope of the straight lines in the log of chromaticity band-ratio space. When the observed values $(\ln(i_{pG}/i_{pR}), \ln(i_{pB}/i_{pR}))^\top$ are projected to the 1-dimensional line perpendicular to the slope, the observed values of the surface points with the same reflectance are projected to a single point as shown in Figure 2(a). Therefore, the distribution of the observed values projected on the 1-D line has some sharp peaks depending on the texture of an under-liquid scene, and then its entropy should be minimal. On the other hand, when they are projected to the 1-D line with the other direction as shown in Figure 2(b), the distribution becomes broader and its entropy should be larger. Hence, we estimate the slope of the straight lines in the log of chromaticity band-ratio space via entropy minimization [6] that is used also for shadow removal from a single image.

Specifically, we denote the slope of the straight line by $(\cos\theta, \sin\theta)^\top$. Then, the projection $i_p(\theta)$ of the observed value $(\ln(i_{pG}/i_{pR}), \ln(i_{pB}/i_{pR}))^\top$ to the direction perpendicular to the slope is given by

$$i_p(\theta) = \ln \frac{i_{pG}}{i_{pR}} \sin\theta - \ln \frac{i_{pB}}{i_{pR}} \cos\theta. \tag{5}$$

We make a normalized histogram with N bins from the projected points on the 1-D line, and compute the Shannon entropy $E(\theta)$ defined as

$$E(\theta) = -\sum_{n=1}^{N} p_n(\theta) \log p_n(\theta). \tag{6}$$

Here, $p_n(\theta)$ is the probability density of the n-th bin.

We minimize the Shannon entropy with respect to the angle θ as

$$\theta = \min_{\hat{\theta}} E(\hat{\theta}) \tag{7}$$

from the interval $[0, \pi]$. We consider the difference of the absorption coefficient is parallel to the slope whose entropy is minimal as

$$\begin{pmatrix} \alpha_G - \alpha_R \\ \alpha_B - \alpha_R \end{pmatrix} // \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}. \tag{8}$$

3.3 Ambiguity

The attenuation due to absorption depends on the product of the absorption coefficient and the depth as shown in eq.(1). Since both the absorption coefficient and the depth are unknown, there is an inherent scale ambiguity between them. Therefore, our proposed method estimates the normalized absorption coefficient such that $\alpha_R + \alpha_G + \alpha_B = 1$. Then, we obtain

$$\alpha_G = \frac{(\sin\theta - 2\cos\theta)\alpha_R + \cos\theta}{\cos\theta + \sin\theta}, \tag{9}$$

$$\alpha_B = \frac{(\cos\theta - 2\sin\theta)\alpha_R + \sin\theta}{\cos\theta + \sin\theta}. \tag{10}$$

Thus, the normalized absorption coefficient estimated by using our method has one degree-of-freedom ambiguity of α_R.

In Section 3.2, we search the angle θ from the interval $[0, \pi]$ because $E(\theta) = E(\theta + \pi)$. Empirically, we can fix the binary ambiguity of θ and $\theta + \pi$ from the color of the captured image; if the image is reddish, $\alpha_G - \alpha_R > 0$ for example. Interestingly, we can resolve the binary ambiguity on the basis of depth recovery if the bias of the depth is fixed as shown in Section 4.1. This is because shape recovery in Section 4.1 yields negative depths (outside the liquid) if the sign of the slope is flipped; it is contradictory to the under-liquid scene.

4 Applications

In this Section, we reveal the effects of the ambiguity of the estimated absorption coefficient on the applications to under-liquid image/scene analysis: shape recovery, absorption removal, and reflectance recovery.

4.1 Shape Recovery

Similar to shape from water [3], we can compute the per-pixel depth of under-liquid scene from eq.(1) as

$$l_p = \frac{1}{2(\alpha_R - \alpha_G)} \left[\ln \frac{i_{pG}}{i_{pR}} - \ln \frac{s_G r_{pG}}{s_R r_{pR}} \right] \tag{11}$$

by using the estimated absorption coefficient with the ambiguity. Note that the overall scale of the depth is unknown because the overall scale (absolute value) of the absorption coefficient is also unknown.

Moreover, the second term of the right-hand side, *i.e.* the bias of the depth is also unknown in our case. In particular, we need to take the difference in per-pixel reflectance $\ln(r_{pG}/r_{pR})$ into consideration although we can ignore $\ln(s_G/s_R)$ because the light source color is uniform across the under-liquid scene. It is in contrast to shape from water [3]; they can assume uniform reflectance with respect to wavelengths because they use two NIR images taken at nearby wavelengths.

To cope with the issue of unknown biases, we use the result of the entropy minimization. Specifically, we find the cluster of pixels with the same reflectance by clustering the projected points on the 1-D line. Then, we estimate the per-class bias via L_1-norm minimization so that the depths of the neighboring pixels are the same. Thus, we can estimate the per-pixel depth of an under-liquid scene up to the ambiguity of an overall scale and an overall bias.

Since the ambiguity has two degrees of freedom, we can fix the scale and bias if the depths of two points are given. Interestingly, we can fix the bias from a surface crossing the border between a liquid and air. Specifically, we can compute the overall bias from eq.(11) because the left-hand side is 0 for surface points outside the liquid. To resolve the ambiguity of the bias is useful for absorption removal in Section 4.2 and for reflectance recovery in Section 4.3. Please see our supplementary material for shape recovery of a dynamic scene.

4.2 Absorption Removal

We can convert the captured image as if the scene were in a transparent liquid without absorption. The pixel value i'_{pc} without absorption is given by

$$i'_{pc} = i_{pc} e^{2\alpha_c l_p} = g_p s_c r_{pc} \tag{12}$$

from eq.(1). Since the attenuation due to absorption depends on the product of the absorption coefficient and the depth, the ambiguity of the scale in the absorption coefficient and in the depth is canceled out each other. Therefore, we can remove the effects of the absorption from the captured image without ambiguity, if the ambiguity in the overall bias of the shape recovery is resolved from surface points outside a liquid.

4.3 Reflectance Recovery

We can compute the per-pixel reflectance of an under-liquid scene as

$$r_{pc} = \frac{i'_{pc}}{g_p s_c} \tag{13}$$

from eq.(12). As discussed in Section 4.2, we can compute the pixel value i'_{pc} with absorption removal. In real-world applications, we can often measure the color of the light source s_c,

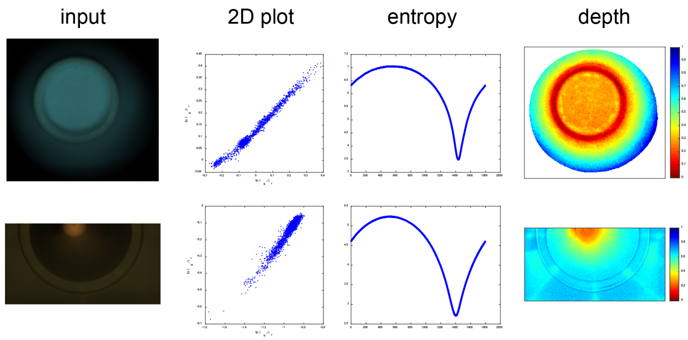

Figure 3: The results of the scenes I (top) and II (bottom): the input image, 2D plot, entropy, and recovered depth from left to right.

but it is difficult to obtain the per-pixel geometric term g_p. Therefore, we can obtain the per-pixel normalized reflectance, *i.e.* $r_{pc}/\sum r_{pc}$ without ambiguity, if the light source color is known and if the ambiguity of the overall bias of the shape recovery is resolved.

5 Experiments

5.1 Setup

To confirm the effectiveness of our proposed method, we conducted a number of experiments using real images. We captured five scenes: I. an upside down cup in diluted methylene blue, II. a colored safety cone in diluted green food coloring, III. a pipe in diluted methylene blue, IV. a wooden board in diluted red food coloring, and V. a textured board in diluted methylene blue. As shown in Figure 1(b), those scenes were illuminated by a halogen lamp and captured with a standard color camera BFS-U3-16S2C-CS from FLIR. We empirically set the number of bins N in Section 3.2 to 256 for the all cases.

5.2 Absorption Coefficient

Figure 3 shows the results of the scenes I (top) and II (bottom): the input image, 2D plot, entropy, and recovered depth from left to right. We can see that the observed values distribute linearly in the log chromaticity band-ratio space, and the entropy has the global minimum. The recovered depths of those scenes normalized to [0,1] qualitatively support the effectiveness of the proposed method. Note that we empirically resolve the ambiguity of θ and $\theta+\pi$ in the slope from the color of the input images.

The estimated absorption coefficients for the five scenes are summarized in Table 1. We measured the ground truth of the absorption coefficients by using a calibration target at known distances. We can see that the estimated absorption coefficients with the ambiguity (w) deviate from the ground truth. For comparison, we fix the unknown parameter α_R of the estimated absorption coefficient by using the ground truth, and re-compute the absorption coefficient without the ambiguity (w/o). We can see that the deviation from the ground truth is explained by the unknown parameter, and therefore we can conclude that our method

Table 1: The estimated absorption coefficients by using our proposed method.

	scene I			scene II		
	GT	w	w/o	GT	w	w/o
α_R	0.60	0.67	0.60	0.53	0.83	0.54
α_G	0.28	0.29	0.30	0.29	0.17	0.27
α_B	0.12	0.04	0.10	0.17	0.00	0.20

	scene III			scene IV			scene V		
	GT	w	w/o	GT	w	w/o	GT	w	w/o
α_R	0.45	0.54	0.45	0.13	0.09	0.14	0.60	0.69	0.59
α_G	0.32	0.30	0.31	0.34	0.31	0.32	0.25	0.26	0.28
α_B	0.23	0.16	0.24	0.53	0.60	0.54	0.15	0.05	0.13

can estimate the three-band absorption coefficient up to one-degree-of-freedom ambiguity.

5.3 Applications

Figure 4 shows the results of the scenes III, IV, and V from top to bottom. Since the pipe in the third scene, the wooden board in the fourth scene, and the textured board in the fifth scene cross the border between liquids and air, we can fix the biases of the depth and resolve the ambiguity of θ and $\theta + \pi$ in the slope. We can confirm that the depth of the pipe outside the liquid is almost zero. In addition, we can see from the result of the scene V that our proposed method works well for a textured surface although there remain minor artifacts around texture edges.

We can see that our absorption removal works well; the color of the input image is different from the ground truth, *i.e.* the color of the object in transparent water without dissolved material, but the color of our absorption removal is similar to the ground truth. The PSNRs below each result image quantitatively show that our absorption removal performs well. We can also see that the normalized reflectance can be recovered well from Figure 4. Thus, we can confirm qualitatively and quantitatively that the estimated absorption coefficient is useful for under-liquid shape recovery, absorption removal, and reflectance recovery even though it has the ambiguity.

6 Conclusion and Future Work

In this paper, we proposed a method for estimating the three-band absorption coefficient of a liquid only from a single color image of an under-liquid scene taken from the outside of the liquid in a passive and non-contact manner. Specifically, our proposed method investigates the observed colors in the log of chromaticity band-ratio space, and estimates the absorption coefficient up to a certain ambiguity via entropy minimization. Moreover, we reveal the effects of the ambiguity on the applications to under-liquid image/scene analysis. We conducted a number of experiments using real images, and confirmed that our method works well and is useful for under-liquid shape recovery, absorption removal, and reflectance recovery. Our future work includes the extension to scattering medium such as underwater vision [1, 2, 4, 8, 9, 16, 17, 22]: the estimation of absorption and scattering coefficients. The integration of spectral imaging with polarimetric imaging [16] and the use of the prior

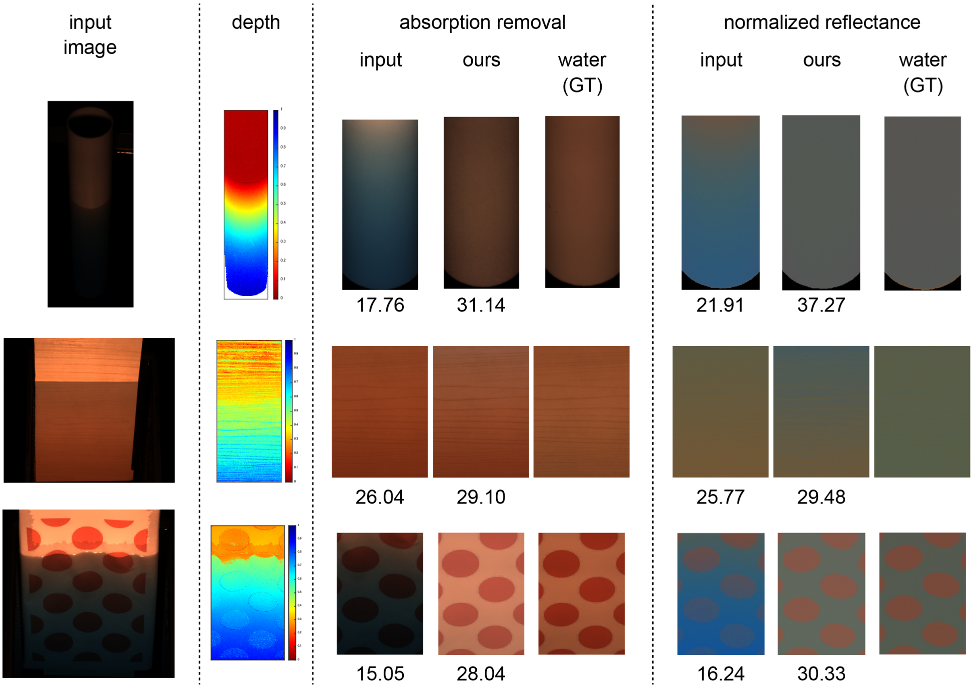

Figure 4: The results of the scenes III, IV, and V from top to bottom: the numerical values below each result image are PSNRs (higher is better).

knowledge with respect to attenuation/absorption coefficients [2] are other directions of our future study.

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